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ABSTRACT

THis is one of eighteen sets of individualized mathematics problems developed by the Oregon Vo-Tech Math Project. Each of these problem packages is organized around a mathematical topic and contains problems related to diverse vocations. Solutions are provided for all problems. This volume includes problems involving logarithms, exponents, and logarithms of trigonometric functions. The problems are drawn from the vocational areas of drafting, forest products, forestry, electronics, clerical work, marketing, and agriculture. (SD)

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I
L INDIVIDUALIZED
LEARNING
S SYSTEMS

Individualized Math Problems in

Logarithms

Oregon Vo-Tech Mathematics Problem Sets

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Individualized Learning Systems

MATHEMATICS

Logarithms

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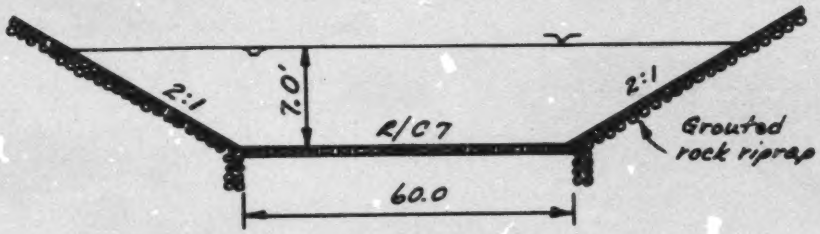
B. Problems without Solutions

1. A restricting factor to flow through a channel is the roughness of the surface with which the water comes in contact. This is denoted as the coefficient of roughness "n". "n" varies with the surface material. Determine an "n" value for the hydraulic design of the channel shown below.



Notes: R/C Reinforced concrete

From Far West states "n" for formed concrete = 0.014

n_1 Grouted rock = 0.030

$$n = \left[\frac{(P_1)(n_1)^{\frac{3}{2}} + (P_2)(n_2)^{\frac{3}{2}}}{P}\right]^{\frac{2}{3}}$$

P_1 = Wetted perimeter of rock

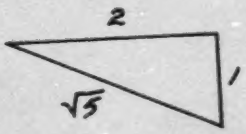
P_2 = Watted perimeter of R/C

P = Total wetted perimeter

Wetted perimeter is the perimeter of the canal with which the water is in contact.

Complete Solutions to B Problems

1.



$\frac{7}{1} = \frac{x}{\sqrt{5}}$ by similar triangles

$x = 15.61$ ft

Wetted perimeters of grouted rock = (2)(15.61 ft) = 31.22 ft
Wetted perimeter of reinforced concrete = 60.00 ft
Total wetter perimeter = 91.22 ft

"n" for formed concrete = 0.014
"n" for grouted rock = 0.030

$$n = \left[\frac{(P_1)(n_1)^{3/2} + (P_2)(n_2)^{3/2}}{P}\right]^{2/3}$$

$$= \left[\frac{(31.22)(0.030)^{3/2} + (60)(0.014)^{3/2}}{91.22}\right]^{2/3}$$

$$= \left[\frac{(31.22)\ \text{antilog}\ (3/2 \log 0.030) + (60)\ \text{antilog}\ (3/2 \log 0.014)}{91.22}\right]^{2/3}$$

$$= \left[\frac{0.1622 + 0.0994}{91.22}\right]^{2/3}$$

$$= \left[\frac{0.2616}{91.22}\right]^{2/3}$$

$$= 0.0028679$$

$$\log n = 2/3 \log [0.0028679]$$

$$= -1.6949527$$

$$n = 0.02018$$

A. Problem with Solution

1. A logger has incurred two debts for equipment he purchased on credit. The principal amount in both cases is for $8,00. however, one note is due at the end of 6 years at 7% compounded annually. The other is due in five years at 5% compounded semiannually. How much cash will he need to pay each of the notes on their due date?

Solution:

A = amount due

$A = P(1 + r)^n$

Where P is the principal, r is the interest rate and n is the number of times the interest is compounded.

a) compounded annually

$A = (\$8,000)(1 + 0.07)^6$
$A = (\$8,000)(1.50073)$
$\log A = \log 8,000 + \log 1.50073$
$\log A = 3.9031 + 0.1763$
$\log A = 4.0794$
$A = \$12,006.67$

b) compounded semiannually

$A = (\$8,000)(1 + 0.05)^{10}$
$A = (\$8,000)(1.62889)$
$\log A = \log 8,000 + \log 1.62889$
$\log A = 3.9031 + 0.2119$
$\log A = 4.1150$
$A = \$13,032.50$

B. Problems without Solutions

2. A small logging company purchases a yarder for $275,000 which includes 7% interest compounded annually. The debt is to be paid in one lump sum at the end of 10 years. How much cash would be required to buy the yarder for cash on the day it was purchased?

2. $P = \frac{A}{(1 + r)^n}$

$P = \frac{\$275,000}{(1 + 0.07)^{10}}$

$\log P = \log 275,000 - 10 \log(1.07)$
$\log P = 5.4393 - 10(0.0293)$
$\log P = 5.4393 - 0.2938$
$\log P = 5.1455$
$P = \$139,800$

3. Let V_o be the original volume
then $V_1 = V_o + (0.08)V_o$
$= V_o(1 + 0.08)$ this is the volume at the end of the first year

$V_2 = V_1(1 + 0.08)$
$= V_o(1 + 0.08)^2$ this is the volume at the end of the second year

Similarly, $V_n = V_o(1 + 0.08)^n$ is the volume at the end of n years

$V_n = 2V_o = V_o(1 + 0.08)^n$
$\log 2 = n \log(1.08)$

$n = \frac{\log 2}{\log 1.08} = \frac{0.3010}{0.0334} = 9.006$ years

A. Problems with Solutions

1 In estimation of forest land value a formula such as the following is used:

$$V_o = \frac{a}{(1 + p)^t - 1}$$ where V_o = land expectation value ($/acre)
a = net income received at rotation age
p = rate of interest
t = length of rotation (years)

a) Determine the land expectation value of a Pine Forest based on a rotation of 60 years, an interest rate of 4% and a net income of $150 at rotation age.

b) Solve the same problem if the interest rate is 7.5% and the net income is $210 at rotation age.

Solution:

a) $$V_o = \frac{\$150}{(1 + 0.04)^{60} - 1}$$

using logarithms to find $x = (1 + 0.04)^{60}$ we have
$\log x = 60 \log 1.04 = (60)(0.01703) = 1.02180$ then
$x = 10.515$.

so,

$$V_o = \frac{\$150}{10.515 - 1} = \frac{\$150}{9.515} = \$15.76/\text{acre}$$

b) $$V_o = \frac{\$210}{(1 + 0.075)^{60} - 1}$$

If $x = (1 + 0.075)^{60}$ then $\log x = (60)(\log 1.075) = (60)(0.03141) = 1.8846$ so, $x = 76.67$ then,

$$\frac{\$210}{76.67 - 1} = V_o$$

$$V_o = \frac{\$210}{75.67} = \$2.77/\text{acre}$$

B. Problems without solutions.

2. Log scaling is subject to individual scalers. The desired accuracy in per cent (E) can be determined by the following formulas:

$$E = 200C\sqrt[3]{\frac{S}{M}}$$

Where c = coeff of variation
S = cost of 100% scaling the average load
M = total sale value

Determine the desired accuracy for :

a) C = 10%; S = \$4.00 ; M = \$40000.

b) C = 7.5%; S = \$2.85 ; M = \$98000.

Complete solutions to B problems.

2. Solution:

Taking the logarithim of each member of the formula

$$\log E = \log 200 + \frac{2}{3}\log C + \frac{1}{3}\left[\log S - \log M\right]$$

a) substituting

$$\log E = \log 200 + \frac{2}{3}\log .10 + \frac{1}{3}\left[\log 4 - \log 40000\right]$$

$$= 2.30103 + \frac{2}{3}\left[9.00000 - 10\right] + \frac{1}{3}\left[0.60206 - 4.60206\right]$$

$$= 2.30103 + \frac{2}{3}\left[29.000 - 30\right] + \frac{1}{3}\left[(10.60206 - 10) - 4.60206\right]$$

$$= 2.30103 + 19.33333 - 20 + \frac{1}{3}\left[26.00000 - 30\right]$$

$$\log E = 1.63436 + 8.66667 - 10 = 10.30103 - 10$$

$$E = 2.00\ \%$$

b) $\log E = 2.30103 + \frac{2}{3}\log .075 + \frac{1}{3}\left[\log 2.85 - \log 98000\right]$

$$= 2.30103 + \frac{2}{3}\left[8.87506 - 10\right] + \frac{1}{3}\left[0.45484 - 4.99123\right]$$

$$= 2.30103 + \frac{2}{3}\left[28.87506 - 30\right] + \frac{1}{3}\left[5.46361 - 10\right]$$

$$= 2.30103 + 19.25004 - 20 + 8.48787 - 10$$

$$\approx .03894$$

$$E = 1.09\%$$

A. Problems with Solutions

1. The equation for the charge Q stored on the plates of a capacitor is:

$Q = CV\left(1 - e^{-\frac{t}{RC}}\right)$ where Q = charge in coulombs; C = capacity in farads; V = voltage applied to the capacitor at t = 0; e = the natural base 2.71826. . . .; t = time in seconds and RC is the product of resistance and capacitance in the series circuit.

(a) Given a circuit where: R = 100 KΩ; C = 0.5 microfarads and V = 325 Volts. Find the charge on the capacitor after 50 Millseconds of time have elapsed from t = 0.

(b) If Q = eV, what voltage is on the capacitor at 50 Millseconds?

Note: 1 Coulomb = 6.24×10 electrons.

Solution:

(a) $Q = CV\left(1 - e^{-\frac{t}{RC}}\right)$

$= (0.5 \times 10^{-6}\text{farads})(325\text{ volts})\left[1 - e^{-\frac{50 \times 10^{-3}\text{ sec}}{(100\times10^{3}\Omega)(0.5\times10^{-6}\text{f})}}\right]$

$= (162.5 \times 10^{-6}\text{coul})(1 - e^{-1})$

$= (162.5 \times 10^{-6}\text{coul})(1 - 0.3679)$

$= 1.027 \times 10^{-4}\text{coul}$

(b) $V = \frac{Q}{C} = \frac{1.027 \times 10^{-4}\text{coul}}{0.5 \times 10^{-6}\text{farads}}$

$= 205.4$ volts

B. Problems without Solutions

2. Power ratios which are sharply non-linear (such as sound levels, power gain, antenna front-to-back ratios, etc.) are measured in terms of the DECIBEL (db). By definition, the decibel is:

$db = 10 \log_{10} \frac{P_2}{P_1}$ where P_1 and P_2 are the power units being compared. This is a very useful relationship in acoustics since a change in sound level or 1 db is just discernable by the ear.

If a high fidelity amplifier is delivering 25 watts of audio power and its output is increased by 3 db, how many watts of power is it now delivering?

Complete Solutions to B Problems

2. $db = 10 \log \frac{P_2}{P_1}$

Let 25 watts = P_1 and the unknown power = P_2

Then, $3 = 10 \log \frac{P_2}{25}$

$0.3 = \text{Log} \frac{P_2}{25}$ Now take antilog of both sides.

$\frac{P_2}{25} = \text{Log}^{-1}\ 0.3 = 2$

$P_2 = 2 \times 25 = 50$ WATTS

$P = 50$ WATTS

Note: For power ratios of any type, a change of 3 db doubles the power, and a change of -3db halves it.

A. Problems with Solutions

1. A fund contains $782.16. If it earns interest at 3% compounded semi-annually (m = 2), how much will be in the fund at the end of 15 1/2 years?

Solution:

Accumulated Sum = Principal invested X (1 + interest rate) raised to the power of the number of intervals or
$S = P(1 + i)^n$ m = number of times interest compounded/year.

$S = (\$782.16)(1 + 0.015)^{31} = (\$782.16)(1.5865)$

$\log S = \log(782.16) + \log(1.5865)$

$\log S = 2.89330 + 0.20044$

$\log S = 3.09374$

$S = \$1,240.90$

2. At what nominal rate compounded semi-annually, will $600 accumulate to $1,150 in 14 years?

Solution:

$(600)(1 + i)^{28} = 1,150$

$(1 + i)^{28} = \frac{1,150}{600}$

$28 \log(1 + i) = \log(1,150) - \log(600)$

$\log(1 + i) = \frac{3.06070 - 2.77815}{28}$

$\log(1 + i) = 0.01009$

$1 + i = 1.0233$

$i = 0.0233$

$j = 2i = 0.0466 = 4.66\%$

A. Problems with Solutions (continued)

3. Maier borrows $8,250 from Boyll which he promises to repay in full at the end of 7 years with 4% accumulated interest. At the end of the fifth year, Boyll sells the note to a bank which charges 5% annual interest, compounded semi-annually. What rate has Boyll actually earned during those five years?

Solution:

Maturity value of note = $(\$8,250)(1.04)^7$
Boyll receives from bank $x, where $x(1.025)^4 = (\$8,250)(1.04)^7$
We need to find at what rate $8,250 accumulates to x in 5 years.

$(8,250)(1 + i)^5 = (8,250)(1.04)^7 (1.025)^{-4}$
$5 \log (1 + i) = 7 \log(1.04) - 4 \log(1.025)$
$5 \log(1 + i) = 7(0.01703) - 4(0.01072)$
$5 \log(1 + i) = 0.07633$
$\log(1 + i) = 0.01526$
$1 + i = 1.0358$
$i = 0.0358 = 3.58\%$

B. Problems without Solutions

4. If money earns 6% interest compounded monthly, how much money should be invested if you will receive $9,643.80 at the end of 24 years?

Solution:

B. Problems without Solutions (continued)

5. What is the necessary interest rate, compounded quarterly, to accumulate $6,394.95 from $2,475 in 22 1/2 years?

   Solution:

6. $3,246.70 earns 5% interest, compounded quarterly. What is the accumulated sum after 14 3/4 years?

   Solution:

7. When must a payment of $3,500 be made to repay a debt of $3,000 which bears interest at 4%, compounded quarterly?

   Solution:

B. Problems without Solutions (continued)

8. After long litigation, a bill of $1,743.20 was settled by a $3,000 payment 15 years after the bill was incurred. What is the equivalent rate of interest compounded semi-annually?

Solution:

9. If money earns 4% annual interest, compounded semi-annually, find the principal if $5,693.60 is to be paid in 8 1/2 years.

Solution:

10. Find the accumulated sum after 7 1/2 years of $439.26 which bears 8 1/2% annual interest, compounded semi-annually.

Solution:

Solutions to B Problems

4. $P = (9,643.80)(1.005)^{-288}$

$\log P = \log(9,643.80) - 288 \log(1.005)$

$\log P = 3.98425 - 288(0.00217)$

$\log P = 3.35929$

$P = \$2,298.20$

5. $\$6394.95 = (\$2,475)(1 + r)^{90}$

$\log(6,394.95) = \log(2,475) + 90 \log(1 + r)$

$$\log (1 + r) = \frac{\log(6,394.95) - \log(2,475)}{90}$$

$$\log (1 + r) = \frac{3.80584 - 3.39358}{90}$$

$\log (1 + r) = 0.00458$

$1 + r = 1.0106$

$r = 0.0106 = 1.06\%$

6. $S = (3,246.70)(1.0125)^{59}$

$\log S = \log(3,246.70) + 59 \log(1.0125)$

$\log S = 3.51144 + 59(0.005395)$

$\log S = 3.829745$

$S = \$6,757.00$

7. $\$3,500 = (\$3,000)(1 + 0.01)^{4n}$

$\log(3,500) = \log(3,000) + 4n \log(1.01)$

$$4n = \frac{\log(3,500) - \log(3,000)}{\log(1.01)}$$

$$4n = \frac{3.54407 - 3.47712}{0.00432}$$

$4n = 15.497685$

$n = 3.87$ years

8. $\$3,000 = (\$1,743.20)(1 + r/2)^{30}$

$\log(3,000) = \log(1,743.20) + 30 \log(1 + r/2)$

$30 \log(1 + r/2) = \log(3,000) - \log(1,743.20)$

$30 \log(1 + r/2) = 3.47712 - 3.24135$

$\log(1 + r/2) = 0.007859$

$1 + r/2 = 1.01827$

$r = 0.03654 = 3.654\%$

Solutions to B Problems (continued)

9. $\$5,693.60 = P(1+0.02)^{17}$
$P = (5,693.60)(1.02)^{-17}$
$\log P = \log(5,693.60) - 17\log(1.02)$
$\log P = 3.755388 - 17(0.00860)$
$\log P = 3.609188$
$P = \$4,066.20$

10. $S = (\$439.26)(1+0.0425)^{15}$
$\log S = \log(439.26) + 15\log(1.0425)$
$\log S = 2.64272 + 15(0.018075)$
$\log S = 2.913845$
$S = \$820.20$

A. Problems with Solutions

1. A fund contains $782.16. If it earns interest at 3% compounded semiannually (m = 2), how much will be in the fund at the end of 15 1/2 years?

Solution:

Accumulated sum = (principal)(1 + periodic interest) raised to a power equal to the number of periodic intervals, or $S = P(1 + i)^n$. m will be used to indicate the number of times interest is computed per year. So,

$$S = P\left(1 + \frac{i}{m}\right)^{(n)(m)}$$

In this problem, P = 782.16

$$\frac{i}{m} = \frac{.03}{2} = .015$$

$$n = (15.5)(2) = 31$$

So, $S = 782.16(1.015)^{31}$

Using logarithms:

$\log S = \log 782.16 + 31 \log 1.015$

$\log S = 2.89330 + .20045$

$\log S = 3.09375$

So, S = $1,240.94

Note: Your answer may differ from this by a few cents if you used tables instead of a calculator.

2. Accumulate $3246.70 for 14 3/4 years at (.05, m = 4)

Solution:

As in problem 1, $S = P\left(1 + \frac{i}{m}\right)^{(n)(m)}$

P = $3,246.70

$$\frac{i}{m} = \frac{.05}{4} = .0125$$

nm = (14.75)(4) = 59

A. Problems with Solutions (continued)

2. (continued)

So, $S = \$3,246.70\ (1 + .0125)^{59}$

$\log S = \log 3,246.70 + 59 \log 1.0125$

$\log S = 3,51144 + 59\ (.00548)$

$\log S = 3.51144 + .31831$

$\log S = 3.82975$

$S = \$6,756.92$

3. If money is worth(.04, m = 2), find the present value today of $5693.60 which is due in 8 1/2 years.

Solution:

$$S = P\left(1 + \frac{i}{m}\right)^{(n)(m)}$$

$S = \$5693.60$

$P = ?$

$\frac{i}{m} = \frac{.04}{2} = .02$

$nm = (8.5)2 = 17$

$\$5693.60 = P\ (1 + .02)^{17}$

$\log 5693.60 = \log P + 17 \log(1 + .02)$

$3.75539 = \log P + (17)(.00860)$

$3.75539 = \log P + .14620$

$3.60918 = \log P$

$\$4,066.16 = P$

B. Problems without Solutions

4. At what rate, compounded quarterly, will $2475 accumulate to $6394.95 in 22 1/2 years?

5. When must a payment of $3500 be made to extinguish a debt of $3000 which bears interest at (.04, m = 4).

B. Problems without Solutions (continued)

6. Accumulate $439.26 for 7 1/2 years at 8 1/2% compounded semiannually.

7. If money is worth 6% compounded monthly (.06, m = 12), find the value today of $9643.80 which is due in 24 years.

8. After long litigation, a bill of $1743.20 was settled by a $3000 payment 15 years after the bill was incurred. What is the equivalent rate of interest compounded semiannually?

B. Problems without Solutions (continued)

9. At what nominal rate, compounded semiannually, will $600 accumulate to $1,150 in 14 years?

10. Maier borrows $8,250 from Boyll which he promises to repay in 7 years with accumulated interest at 4%. Five years from now, Boyll sells the note to a bank which charges (.05, m = 2). What rate has Boyll actually earned during those 5 years?

Complete Solutions to B Problems

4. $S = P\left(1 + \frac{i}{m}\right)^{nm}$ $S = \$6,394.95$

$P = \$2,475$

$\frac{i}{m} = ?$

$m = 4$

$mn = (22.5)(4) = 90$

$\$6,394.95 = \$2,475\left(1 + \frac{i}{m}\right)^{90}$

$\log 6,394.95 = \log 2,475 + 90 \log\left(1 + \frac{i}{m}\right)$

$3.80584 = 3.39358 + 90 \log\left(1 + \frac{i}{m}\right)$

$.41226 = 90 \log\left(1 + \frac{i}{m}\right)$

$.00458 = \log\left(1 + \frac{i}{m}\right)$

$1.01060 = 1 + \frac{i}{m}$

$.01060 = \frac{i}{m}$

$4(.01060) = i$

$.04241 = i$

$i = 4.24\%$

5. $S = P\left(1 + \frac{i}{m}\right)^{nm}$

$S = \$3,500$ $P = \$3,000$ $\frac{i}{m} = .01$ $mn = 4(?)$

$\$3,500 = \$3,000\ (1 + .01)^{4n}$

$\log 3,500 = \log 3,000 + 4n \log (1 + .01)$

$3.54407 = 3.47712 + 4n\ (.00432)$

Complete Solutions to B Problems (continued)

5. (continued)

$3.54407 - 3.47712 = 4n\ (.00432)$

$.06695 = 4n\ (.00432)$

$.06695 = n\ (.01729)$

$\frac{.06695}{.01729} = n$

$3.87 \text{ yrs} = n$

6. $S = P\left(1 + \frac{i}{m}\right)^{nm}$

$S = ?\quad P = \$439.26$

$\frac{i}{m} = \frac{.085}{2} = .0425$

$mn = (2)(7.5) = 15$

$S = \$439.26\ (1 + .0425)^{15}$

$\text{Log } S = \log 439.26 + 15 \log\ (1 + .0425)$

$\text{Log } S = \log 439.26 + 15 \log 1.0425$

$\text{Log } S = 2.64272 + 15\ (.01808)$

$\text{Log } S = 2.64272 + .27114$

$\text{Log } S = 2.91386$

$S = \$820.09$

7. $S = P\left(1 + \frac{i}{m}\right)^{nm}$

$S = \$9,643.80 \quad P = ? \quad \frac{i}{m} = \frac{.06}{12} = .005 \quad mn = (12)(24) = 288$

$\$9,643.80 = P\ (1 + .005)^{288}$

Complete Solutions to B Problems (continued)

7. (continued)

$\log 9{,}643.80 = \log P + 288 \log(1 + .005)$

$\log 9{,}643.80 = \log P + 288 \log(1.005)$

$3.98425 = \log P + 288(.00217)$

$3.98425 = \log P + .62383$

$3.98425 - .62383 = \log P$

$3.36042 = \log P$

$\$2{,}293.11 = P$

8. $S = P\left(1 + \frac{i}{m}\right)^{nm}$

$S = \$3{,}000 \quad P = \$1{,}743.20 \quad \frac{i}{m} = ? \quad m = 2 \quad nm = (2)(15) = 30$

$\$3{,}000 = \$1{,}743.20\left(1 + \frac{i}{2}\right)^{30}$

$\log 3{,}000 = \log 1{,}743.20 + 30 \log\left(1 + \frac{i}{2}\right)$

$3.47712 = 3.24135 + 30 \log\left(1 + \frac{i}{2}\right)$

$.23577 = 30 \log\left(1 + \frac{i}{2}\right)$

$.00785 = \log\left(1 + \frac{i}{2}\right)$

$1.01826 = 1 + \frac{i}{2}$

$.01826 = \frac{i}{2}$

$.03652 = i \qquad .0365 = 3.65\%$

Complete Solutions to B Problems (continued)

9. $S = P\left(1 + \frac{i}{m}\right)^{mn}$

$S = \$1,150 \quad P = \$600 \quad \frac{i}{m} = ? \quad mn = (2)(14) = 28$

$\$1,150 = \$600\left(1 + \frac{i}{2}\right)^{28}$

$\log 1,150 = \log 600 + 28 \log\left(1 + \frac{i}{2}\right)$

$3.06070 = 2.77815 + 28 \log\left(1 + \frac{i}{2}\right)$

$.28255 = 28 \log\left(1 + \frac{i}{2}\right)$

$.01009 = \log\left(1 + \frac{i}{2}\right)$

$1.02351 = 1 + \frac{i}{2}$

$.02351 = \frac{i}{2}$

$.04702 = i \qquad .04702 = 4.70\%$

10. Another way to write the formula $S = P(1 + i)^n$ is $P = S(1 + i)^{-n}$. $(1 + i)^{-n}$ is referred to as the discount factor and $(1 + i)^n$ as the accumulation factor. When we want the present value of an amount of money, we just multiply it by $(1 + i)^{-n}$.

In this problem, the maturity value of the note is $\$8,250(1.04)^{-1}$. Of course, the bank does not pay that much to Boyll for the note, but must "discount" the note for the two years till maturity. Let x = dollars bank pays Boyll. Then $x = [8250(1.04)^7] \times [(.025)^{-4}]$. To find the interest rate Boyll earns, we need to find the rate necessary to accumulate \$8,250 to x dollars in 5 years.

Thus, $8,250(1 + i)^5 = 8,250(1.04)^7 (1.025)^{-4}$
Before taking logs, divide both sides by 8,250. Then,

$5 \log(1 + i) = 7 \log(1.04) - 4 \log(1.025)$

$= 7(.017033) - 4(.010724)$

$= .0763379$

Complete Solutions to B Problems (continued)

10. (continued)

So, $\log(1 + i) = .01527$

$1 + i = 1.0358$

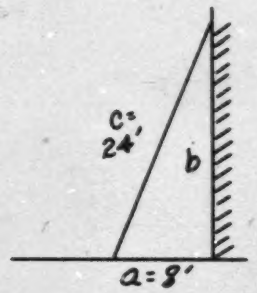
$i = 3.58\%$

This one may give you a little difficulty, so study it carefully and slowly.

A. Problems with solutions.

1. The foot of a 24 ft. ladder is 8 ft. from the wall of a building against which its top rests. How high does the ladder reach on the wall?

Solution:



$a^2 + b^2 = c^2$

$(8\ ft.)^2 + (b)^2 = (24\ ft.)^2$

$64\ ft.^2 + b^2 = 576\ ft^2$

$b^2 = 576\ ft.^2 - 64\ ft.^2 = 512\ ft.^2$

$b = 22.63\ ft.$

2. One method for finding the area of a triangle is to use the formula:

$$A = \sqrt{(s)\ (S-a)\ (S-b)\ (S-c)}$$

in which S equals one-half the perimeter of the triangle, and a, b, and c are lengths of the three sides. Find the number of area in a triangular field with sides 1,024.6 ft., 2,104.3 ft. and 2,359.9 ft.

Solution:

$$S = \frac{1024.6\ ft. + 2104.3\ ft. + 2359.9\ ft.}{2} = \frac{5488.8\ ft.}{2} = 2744.4\ ft.$$

$$A = \sqrt{\frac{(2744.4ft.)(2744.4 - 1024.6\ ft)\ (2744.4 - 2104.3)ft\ (2744.4 - 2359.9)\ ft}{43.560\ ft.^2/\ acre}}$$

$$A = \sqrt{\frac{(2744.4ft.)(1719.8\ ft.)\ (640.1\ ft.)\ (384.5\ ft.)}{43.560\ ft.^2/\ acre}}$$

$\log 2744.4 = 3.43845$

$+\log 1719.8 = 3.23548$
$+\log\ 640.1 = 2.80625$
$+\log\ 384.5 = 2.58490$

$\log R = 12.06508$

$2 = 6.03254$

$-\log 43,560 = 4.63909$

$\log A = 1.39345$

$A = 24.743\ acres$

A. Problems with solutions, continued.

3. One method for finding the area of a triangle is to use the formula:

Area = 1/2 (side)(side)(sine of included angle)

Find the area in acres, of a triangular field having one side that is 1824 ft. long, one that is 834 ft. long and the sides form an angle of 52°.

Solution:

$$\text{Area} = \frac{\frac{(834 \text{ ft.})\ (1824 \text{ ft.})\ (\sin 52^\circ)}{2}}{\frac{43{,}560 \text{ ft.}^2}{\text{acre}}}$$

log 834	= 2.92117
log 1824	= 3,26102
+ log sin 52°	= 9.89653 - 10
log N	= 6.07872
- log 2	= 0.30103
log A in ft.	= 5.77769
-log 43560	= 4.63909
log A in acres	= 1.13860

Area+ 13.76 acres

B. Problems without solutions.

4. How many feet long is the radius of a circle whose area is 10 acres?

5. Find the number of acres in a triangular field with sides: 512.4 ft., 602.4 ft., and 600.0 ft.

B. Problems without solutions.

6. Find the area in square ft. of a triangular field having sides of 42.3 ft., 27.1 ft. and the sides form an angle of 21.5°.

7. How many ton of wheat are there in a conical pile measuring 80 ft. high and having a base diameter of 68.5 ft? (assuming a weight of 48 lbs/ft^3.)

8. At 5,255 gallons per minute, how many acre feet of water can be pumped in 24 hours?

B. Problems without solutions.

9. Determine the volume in gallons of storage for a spherical tank 20.3 ft. in diameter.

10. If you apply 5,875 gallons per minute, how many hours will it take to apply 1 in. of water to 154.8 acres?

B. Problems without solutions.

11. A formula for the velocity (feet per second) of water in a drain tile is given as:

$$V = 138\ R^{2/3}\ S^{1/2}$$

where S is the slope and R depends upon the size of the tile and the depth of the water running in it. For a 6-in. tile running about full, R = 0.135. Under these conditions, what would be the velocity of the water in a tile line whose slope is 0.15%?

12. The soil on your farm has an infiltration rate $I = 3.5\ T^{-.64}$ where I is the instantaneous rate in inches per hour and T is the time in minutes since irrigation began. Using logarithms determine the following information:

a) The infiltration rate two hours after irrigation began.

b) The time when the rate will have dropped to 0.3 in./hr., which is the rate at which runoff from the sprinkler will begin.

Complete solutions to B problems.

4. Area $= \pi r^2$

$$(10 \text{ acres}) \left(\frac{43{,}560 \text{ ft.}}{1 \text{ acre}}\right) = 435{,}600 \text{ ft.}^2$$

$$435{,}600 \text{ ft.}^2 = (3.1416)\ (r)^2$$

$$\frac{435{,}600 \text{ ft.}^2}{3.1416} = r^2$$

$$138655.46 \text{ ft.}^2 = r^2$$

$$372.4 \text{ ft} = r$$

5. $$S = \frac{512.4 \text{ ft} + 602.4 \text{ ft.} + 600.0 \text{ ft.}}{2} = \frac{1714.8 \text{ lbs}}{2} = 857.4 \text{ lbs.}$$

$$A = \sqrt{\frac{(857.4 \text{ ft.})\ (857.4 - 512.4)\text{ft.}\ (857.4 - 602.4)\text{ft.}\ (857.4 - 600.0)\text{ft.}}{43{,}560 \text{ ft}^2/\text{acre}}}$$

$$A = \sqrt{\frac{(857.4 \text{ ft})\ (345.0 \text{ ft.})\ (255.0 \text{ ft})\ (257.4 \text{ ft})}{43{,}560 \text{ ft}^2/\text{ acre}}}$$

$\log 857.4 = 2.93318$
$\log 345.0 = 2.53782$
$\log 255.0 = 2.40654$
$\log 257.4 = 2.41061$

$\log R = 10.28815$
$\div 2 = 5.14408$
$-\log 43560 = 4.63909$

$\log A = 0.50499$
$A = 3.199$ acres

6. $$\text{Area} = \frac{(42.3 \text{ ft})\ (27.1 \text{ ft})(\sin 21.5^\circ)}{2}$$

$\log 42.3 = 1.62634$
$\log 27.1 = 1.43297$
$\log \sin 21.5^\circ = 9.56408 - 10$

$\log N = 2.62339$
$-\log 2 = 0.30103$

$\log A = 2.32236$
$A = 210 \text{ ft}^2$

Complete solutions to B problems, continued.

7. $V = 1/3\ r^2 h \pi$

$W = (1/3)\ (3.1416)(34.25\ \text{ft.})^2\ (80\ \text{ft.})\ \left(\frac{48\ \text{lbs.}}{\text{ft}^3}\right)\ \left(\frac{1\ \text{ton}}{2{,}000\ \text{lbs.}}\right)$

log 3.1416	= 0.49715	
+2 log 34.25	= 3.06932	
+ log 80	= 1.90309	
+ log 48	= 1.68124	
log N	= 7.15080	
log D	= 3.77815	
log W	= 3.37265	
W	= 2358.6 ton or approx. 2,360 ton	

log 3	= 0.47712
+log 2000	= 3.30103
log D	= 3.77815

8. $\text{No.} = \left(\frac{5{,}255\ \text{gal.}}{\text{min.}}\right)\ \left(\frac{1\ \text{ft.}^3}{7.5\ \text{gal.}}\right)\ \left(\frac{1\ \text{acre}}{43{,}560\ \text{ft.}^2}\right)\ \left(\frac{60\ \text{min.}}{1\ \text{hr.}}\right)\ (24\ \text{hr.})$

log 5,255	= 3.72057	
+log 60	= 1.77815	
+ log 24	= 1.38021	
log N	= 6.87893	
log D	= 5.51415	
log No.	= 1.36478	
No.	= 23.16 acre feet/24 hr.	

log 7.5	= 0.87506
+log 43569	= 4.63909
log D	= 5.51415

9. $V = 4/3\pi r^3$

$(4/3)(3.1416)\ (10.15\ \text{ft.})^3\ \left(\frac{7.5\ \text{gal.}}{1\ \text{ft.}^3}\right)$

log 4	= 0.60206	
+ log 3.1416	= 0.49715	
+ 3 log 10.15	= 3.01941	
+ log 7.5	= 0.87506	
log N	= 4.99368	
log 3	= 0.47712	
log V	= 4.51656	
V	= 32851 gals. or approx. 32,900 gals.	

Complete solutions to B problems, continued.

10. $A = (154.8 \text{ acres}) \left(\frac{43{,}560 \text{ ft.}^2}{1 \text{ acre}}\right) \left(\frac{1 \text{ ft.}}{12}\right) \left(\frac{7.5 \text{ gals.}}{1 \text{ ft.}^3}\right) \left(\frac{1 \text{ min}}{1780 \text{ gal}}\right) \left(\frac{1 \text{ hr}}{60 \text{ min.}}\right)$

log 154.8	= 2.18977	log 12	= 1.07918
+log 43560	= 4.63909	log 1780	= 3.25042
+log 7.5	= 0.87506	log 60	= 1.77815
log N	= 7.70392	log D	= 6.10775
log D	= 6.10775		
log A	= 1.59617		

A = 39.46 hours or approx. 40 hours.

11. $V = (138)(0.135)^{2/3}(0.0015)^{1/2}$

log 138	= 2.13988
+2/3 log 0.135	= 9.42022 - 10
+1/2 log 0.0015	= 8.58805 - 10
log V	= 20.14815 - 20
V	= 1.406 ft/ sec.

12. a) $I = 3.5\ (120)^{-.64}$

$I = \frac{3.5}{(120)^{.64}}$

log 3.5	= 10.54407 - 10
-.64 log 120	= 1.33068
log I	= 9.21339 - 10
I	= .1630

b) $0.3 = 3.5\ T^{-.64}$

$\frac{0.3}{3.5} = \frac{1}{T^{.64}}$

$\frac{3.5}{0.3} = T^{.64}$

$\sqrt[.64]{\frac{3.5}{0.3}} = T$

log 3.5	= 10.54407 - 10
-log 0.3	= 9.47712 - 10
log R	= 1.06695
÷ .64	= 1.66711
T	= 46.46 min. or 46.5 min.